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THE EFFECT OF SINGLE-PARTICLE CHARGE LIMITS
ON CHARGE DISTRIBUTIONS IN DUSTY PLASMAS

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Abstract

An analytical expression for the stationary particle charge distribution in dusty plasmas is derived that accounts for the existence of single-particle charge limits. This expression is validated by comparison with the results of Monte Carlo charging simulations. The relative importance of the existence of charge limits for various values of the ratio of electron-to-ion density and ion mass is examined, and the effect of charge limits on the transient behavior of the charge distribution is considered. It is found that the time required to reach a steady-state charge distribution strongly decreases as the charge limit decreases, and that the existence of charge limits causes high-frequency charge fluctuations to become relatively more important than in the case without charge limits.

1. Introduction

The stochastic nature of particle charging in plasmas causes a population of dust particles of given size to exhibit a distribution of charge states. A number of previous investigators have analyzed the form of this distribution. However none of these studies accounted for the fact that the amount of charge a dust particle can hold is limited. On the other hand, several studies involving numerical modeling of particle formation and growth in plasmas did account for the existence of particle charge limits [1-5], but these studies did not address the more general question of how charge limits affect charge distributions.

The primary purpose of the present work is thus to examine the effect of charge limits on particle charge distributions in dusty plasmas. Assuming that charging is dominated by collisional processes (i.e., attachment of electrons and ions by collisions with dust particles), the higher mobility of electrons than of ions causes dust particles in plasmas under most conditions to be predominantly negatively charged. Hence we here consider only negative charge limits, in particular the maximum number of electrons a particle can hold, and we focus on particles with sizes in the nanoscale regime, where stochastic charging is most important and the existence of charge limits is likely to be most consequential.

Cui and Goree used a Monte Carlo (MC) model to analyze charge fluctuations on dust particles in a plasma [6]. They showed that the root-mean-square charge fluctuation varies as \( dq = 0.5(\bar{q})^{-1/2} \), where \( \bar{q} \) is the mean charge, and that power spectra of the fluctuations are dominated by low (~kHz) frequencies. Stochastic charging was described by Matsoukas and Russell
as a one-step Markov process whose continuous form could be cast as a Fokker-Planck differential equation where the convective and diffusive terms are functions of electron and ion currents to the particle [7, 8]. Assuming that particle charging follows orbital motion limited (OML) theory [9], they showed that in the stationary case, where electron and ion currents to dust particles balance each other, the charge distribution has a Gaussian form, provided that the average charge is not too small. Shotorban extended the analytical solution to the nonstationary case [10]. The dynamic behavior of particle charge fluctuations was studied by Khrapak et al., who added the effects of thermonic and photoelectric emission from particles [11]. Gordiets and Ferreira studied the effect on charge distributions of secondary electron emission [12]. Schaffer and Burns studied stochastic charge fluctuations in planetary rings [13], and Draine and Sutin studied charge distributions on interstellar grains [14]. They noted the existence of charge limits due to electron field emission, but did not account for these in their analysis of charge distributions.

Here we derive an analytical expression for the stationary particle charge distribution that accounts for the existence of charge limits, and compare this expression with numerical simulations that employ a Monte Carlo charging model. Results are shown for typical low-pressure plasma conditions. As a particle’s charge limit depends only on its size and the material of which it is composed, rather than on the plasma environment, results can be extended to a variety of plasma conditions. Dust particles often deplete electrons in plasmas sufficiently that the electron density $n_e$ becomes much lower than the positive ion density $n_i$. We show that the extent to which the existence of charge limits affects charge distributions depends strongly on the ratio $n_e / n_i$. Additionally, we consider the effect of ion mass on the importance of charge limits, and consider the effect of charge limits on the temporal behavior of particle charge distributions, including both the time required to reach a steady-state distribution and the power spectrum of charge fluctuations.

2. Review of single-particle charge limits

We first briefly review expressions in the literature for the values of charge limits due to various pertinent effects.

The finite surface tension of liquid droplets imposes a limit, known as the Rayleigh limit, to the number of charges that a droplet can hold. As the charge increases, Coulomb repulsion finally becomes greater than the surface tension, causing the droplet to break up. Here we focus on
solid particles, for which the surface tension is much higher than for liquids, and other phenomena impose more stringent charge limits.

The main phenomena that limit the number of electrons that a solid particle can hold are electron field emission, which is independent of particle material, and material-dependent electron affinity.

Electron field emission is the spontaneous emission of electrons from a negatively-charged particle if the self-generated electric field at the particle surface exceeds a critical value. This value is on the order of \( \sim 10^7 \) V cm\(^{-1}\) [15], for which Draine and Sutin determined that the maximum number \(|q_E|\) of negative charges a particle can hold is given by

\[
|q_E| = 1 + 0.7 \left( \frac{R}{1 \text{ nm}} \right)^2 ,
\]

where \(R\) is the particle radius [14].

The work required to remove an electron from a particle, i.e. its electron affinity, is affected by surface curvature as well as by Coulomb repulsion due to pre-existing negative charges on the particle. Thus a charged spherical nanoparticle with \(|q|\) electrons attached has an effective electron affinity that differs from the value for the bulk (flat) neutral material, and this imposes another charge limit, determined by the condition that for an additional electron to attach to the particle the electron affinity cannot be negative.

Accounting for these effects, the electron affinity \(A\) of a particle is given by

\[
A = A_\infty - \frac{5}{8} \frac{e^2}{\pi \varepsilon_0 R} - \frac{(|q|-1)e^2}{4\pi \varepsilon_0 R} ,
\]

where \(A_\infty\) is the electron affinity of the bulk neutral material [16].

The corresponding charge limit \(|q_A|\) is obtained by setting \(A\) to zero, giving

\[
|q_A| = \left( \frac{4\pi \varepsilon_0 A_\infty}{e^2} \right) R + \frac{3}{8} .
\]
More detailed models have been proposed in the literature for the effective electron affinity and corresponding charge limit of semiconductor nanoparticles, that introduce a dependence on the relative permittivity of the particle material [1, 17].

Figure 1 shows negative charge limits for silicon nanoparticles \( (A_\infty = 4.05 \text{ eV}) \) given by equations (1) and (3) as well as by Gallagher’s model [1], in which the charge limit is proportional to \( R^{3/2} \). (Ref. [17] gives a result for silicon that is virtually identical to equation (3).) As charge is an integer quantity, these expressions are all converted here to integer form. That is,

\[
\text{if } Z \leq |q_{E,A}| < Z + 1, \text{ then } q_{\text{lim}} = Z, \tag{4}
\]

where \( Z \) is a non-negative integer and \( |q_{E,A}| \) is the magnitude of the charge limit given by equation (1), (3), or some other suitable expression.

Taking equation (3) as the governing expression, one notes that for particles of given size \( Q_{\text{lim}} \propto A_\infty \), except for cases where the charge limit is small enough that the term \( 3/8 \) contributes

![Figure 1. Charge limits for silicon nanoparticles based on electron field emission, equation (1), and particle electron affinity given either by equation (3) or Ref. [1], all shown in integer form.](image)
significantly. For example, the bulk electron affinity of SiO\textsubscript{2} equals 1.0 eV, approximately four times smaller than the value for pure Si. Therefore the charge limit of SiO\textsubscript{2} particles would be four times smaller (or corresponding integer value given by equation (4)) than for Si, for particles of the same size.

3. Analytical expression for particle charge distribution function accounting for existence of charge limits

We assume that dust particles are charged only by electron and ion attachment, and we apply OML theory, which assumes that dust particle radii are much smaller than the plasma Debye length [9]. Additionally we assume that particles are electrically isolated from each other. Under some conditions one can expect these assumptions to be violated. Thus the work presented here can be taken as a correction to previous work that makes these same assumptions but which neglects the existence of charge limits. A more accurate description under some conditions will require that one accounts for additional charging mechanisms, deviations from OML theory for electron and ion currents to dust particles, and electrostatic interactions of dust particles with each other.

OML theory gives continuous expressions for electron ($I_e$) and ion ($I_i$) currents to particles as a function of particle charge $q$. For Maxwellian velocity distributions, these currents are given by

$$I_x(q) = \begin{cases} I_{x0} \exp \left( \frac{qxe^2}{4\pi\varepsilon_0 RkT_x} \right), & q \geq 0 \\ I_{x0} \left( 1 - \frac{qxe^2}{4\pi\varepsilon_0 RkT_x} \right), & q < 0 \end{cases},$$

(5)

where

$$I_{x0} = \pi R^2 s_x n_x \sqrt{\frac{8kT_x}{\pi m_x}}.$$  

(6)

Here $x \equiv e,i$ for electrons and ions, respectively, with $q_x$ representing charge ($q_e = -1$, $q_i = +1$), $s_x$ sticking coefficient, $T_x$ temperature, $m_x$ mass, $n_x$ number density, $k$ the Boltzmann constant, $e$ the elementary charge, and $\varepsilon_0$ the vacuum permittivity [18].
Based on these currents, Matsoukas and Russell derived an analytical solution for the stationary particle charge distribution under the assumptions that $4\pi\varepsilon_0 R^2 / e^2 \ll 1$ and that the average particle charge is not too small [7, 8]. They showed that the distribution function for particle charge can be approximated by a normalized Gaussian distribution,

$$n(q) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{q - \bar{q}}{\sigma} \right)^2 \right], \quad (7)$$

where $\bar{q}$ is the average particle charge, $\sigma$ is the standard deviation of the distribution, and

$$\int_{-\infty}^{\infty} n(q) dq = 1. \quad (8)$$

For Maxwellian electron and ion velocity distributions,

$$\bar{q} = C \frac{4\pi\varepsilon_0 R k T_e}{e^2} \ln \left[ \frac{s_n}{s_p} \left( \frac{m_i T_e}{m_e T_i} \right)^{1/2} \right], \quad (9)$$

and

$$\sigma^2 = \frac{4\pi\varepsilon_0 R k T_e}{e^2} \left( 1 - \frac{T_e}{T_e + T_i} - \frac{e^2}{4\pi\varepsilon_0 R k \bar{q}} \right), \quad (10)$$

$\sigma^2$ being the variance of the distribution. In equation (9), $C$ is a constant that is weakly dependent on plasma parameters and independent of particle size. For argon plasmas over a wide range of temperatures, $C = 0.73$ [7].

The Gaussian particle charge distribution defined by equations (7)-(10) does not account for the existence of particle charge limits, and thus it assigns a non-zero probability to a particle’s having a charge that is more negative than its charge limit ($-q_{lim}$). Here we propose a correction to equations (7)-(10) to account for the existence of charge limits.
To obtain an expression for the corrected charge distribution $n^*(q)$ we renormalize $n(q)$ with the negative charge limit as a lower bound on $q$. Let the correction factor $\xi$ be defined by the relation

$$\xi \int_{q^\prime_{\lim}}^{\infty} n(q) dq = 1 \quad (11)$$

where the lower limit of integration $q^\prime_{\lim}$ is given by

$$q^\prime_{\lim} = -q_{\lim} - 0.5 \quad (12)$$

and $n(q)$ is the uncorrected particle charge distribution function given by equation (7).

The term 0.5 in equation (12) corrects for the fact that the number of charges on a particle is treated in equation (11) as a continuous variable whereas in reality it is integer. Thus each integer charge $Q$ is centered within a charge interval of width equal to unity, and the probability that a particle has charge $Q$ is given by

$$\int_{Q-1/2}^{Q+1/2} n^*(q) dq \quad (13)$$

Solving equation (11) using integral $I_1$ in the Appendix, the correction factor is found to be given by

$$\xi = \frac{2}{1 + \text{erf} \left( \frac{q - q^\prime_{\lim}}{\sqrt{2}\sigma} \right)} \quad (14)$$

Thus the corrected particle charge distribution function, accounting for the existence of charge limits, can be written as
\[ n^*(q) = \frac{2}{1 + \text{erf}\left(\frac{q - q'_{\text{lim}}}{\sqrt{2}\sigma}\right)} H(q - q'_{\text{lim}}) n(q), \quad (15) \]

where \( n(q) \) is given by equation (7), \( \bar{q} \) by equation (9), \( q'_{\text{lim}} \) by equation (12), \( \sigma \) is obtained from equation (10), and \( H \) is the Heaviside step function.

The corresponding average particle charge and variance, for the distribution accounting for the existence of charge limits, are then obtained from:

\[ \bar{q}^* = \int_{q'_{\text{lim}}}^{\infty} q n^*(q) dq \quad (16) \]

and

\[ \sigma^*^2 = \int_{q'_{\text{lim}}}^{\infty} (q - \bar{q}^*)^2 n^*(q) dq. \quad (17) \]

If the magnitude of the charge limit is much greater than the average number of negative charges in the absence of charge limits (\( q'_{\text{lim}} \gg |\bar{q}| \)), then the existence of the charge limit barely constrains the charge distribution. This is confirmed by inspection of equations (14) and (15), which indicates that in this regime the existence of charge limits has negligible effect: \( \xi \rightarrow 1 \) and \( n^*(q) \rightarrow n(q) \).

The average charge and variance in the case of charge limits can be written

\[ \bar{q}^* = \bar{q} + \Delta q \quad (18) \]

and

\[ \sigma^*^2 = \sigma^2 + \Delta(\sigma^2), \quad (19) \]

where \( \Delta q \) and \( \Delta(\sigma^2) \) are corrections that must be made to the average particle charge and variance to account for the existence of charge limits.

Using integrals \( I_2 \) and \( I_3 \) in the Appendix, respectively, we obtain
\[
\Delta q = \frac{1}{\sqrt{2\pi}} \frac{2\sigma}{1 + \text{erf}\left(\frac{\bar{q} - q'_\text{lim}}{\sqrt{2}\sigma}\right)} \exp \left[-\frac{1}{2}\left(\frac{\bar{q} - q'_\text{lim}}{\sigma}\right)^2\right],
\]

(20)

and

\[
\Delta (\sigma^2) = (\Delta q)^2 + (\bar{q} - 2\bar{q}^* + q'_\text{lim}) \Delta q.
\]

(21)

4. Test of validity of analytical expressions

As a test of these analytical expressions, we conducted MC simulations for 5- and 10-nm-diameter particles in an argon plasma \((m_e / m_i = 1.37 \times 10^{-5})\) with \(n_e / n_i = 1\), \(T_e = 3\) eV, and \(T_i = 300\) K. The MC model is the same as used by Cui and Goree [6], except that the electron sticking coefficient \(s_e\) is set to zero when the magnitude of the (negative) charge equals the charge limit. Particle composition was unspecified; instead the value of the charge limit was taken as a free parameter.

Figure 2 shows comparisons of the analytical expression for the charge distribution, equation (15), with MC simulations that each involved \(2 \times 10^7\) collisions of electrons and ions with a nanoparticle. Figure 2(a) shows results for 5-nm-diameter particles and Fig. 2(b) for 10-nm-

![Figure 2](image-url)

Figure 2. Comparison of analytical expression, equation (15) (solid lines), with Monte Carlo simulations (histograms) for argon plasma with \(n_e / n_i = 1\), \(T_e = 3\) eV, \(T_i = 300\) K. Particle diameter = (a) 5 nm, (b) 10 nm.
diameter particles. For particles of each size the top graph shows the case without charge limits (denoted $q_{\text{lim}} = \infty$); the middle graph shows a case where $q_{\text{lim}}$ is somewhat greater than the magnitude of the average charge $\bar{q}$ without charge limits; and the bottom graph shows a case where $q_{\text{lim}}$ is much smaller than the magnitude of $\bar{q}$. The analytical expression is seen to agree quite well with the MC simulations for all cases shown. Note that the results with charge limit do not represent a simple truncation of the Gaussian distribution without charge limits, but a truncation together with a redistribution of particles into charge states, according to equation (15).

Another way to characterize the agreement between the analytical expression and the MC simulations is to compare the average charge and standard deviation predicted by the two methods. These comparisons, for 10-nm-diameter particles, for which the average charge $\bar{q}$ without charge limits equals −24.5, are shown in Figs. 3 and 4. Accounting for the existence of charge limits the results for average charge (Fig. 3) show excellent agreement between the analytical expression and MC simulations over the entire range of values of charge limit shown.

Figures 3 and 4 also show that for values of $q_{\text{lim}}$ greater, in this case, than about 30, the average charge and standard deviation become independent of the value of the charge limit. In this regime, where the charge limit is significantly greater than the average charge $\bar{q}$ without charge limits, the existence of charge limits has only a small effect on the charge distribution—both av-

![Figure 3](image1.png)  
**Figure 3.** Average particle charge as a function of charge limit for 10-nm-diameter particles in an argon plasma with $n_e/n_i = 1$, $T_e = 3$ eV, $T_i = 300$ K.

![Figure 4](image2.png)  
**Figure 4.** Standard deviation of charge distribution as a function of charge limit for 10-nm-diameter particles in an argon plasma with $n_e/n_i = 1$, $T_e = 3$ eV, $T_i = 300$ K.
average charge and standard deviation are virtually unaffected by the existence of charge limits. However, as the value of $q_{\text{lim}}$ is reduced so that it is of the same order or smaller than $\bar{q}$, the effect of charge limits becomes increasingly apparent.

With respect to the standard deviation of the charge distribution, Fig. 4, the analytical expression agrees well with the MC simulations for values of $q_{\text{lim}}$ greater than about 20, but then deviates increasingly as $q_{\text{lim}}$ is decreased. This discrepancy, while small in absolute terms—it is much less than the elementary charge $e$—results from the fact that as the average charge approaches zero the assumptions underlying the derivation of equation (7) are violated, and the charge distribution is expected to deviate from a Gaussian form, as discussed in Appendix B of Ref. [7]. The MC simulations, of course, make no assumption regarding the form of the charge distribution.

### 5. Effect of electron depletion

The depletion of electrons by attachment to dust particles can cause the electron density to become much lower than the positive ion density. For example, recent experiments measured a three-fold depletion in electron density relative to ion density in an argon-silane dusty plasma [19], and recent self-consistent numerical simulations of dusty plasmas indicate that under some circumstances the positive ion density in the bulk plasma can exceed the electron density by up to a factor of several hundred [4]. Interestingly, this feature can mitigate the extent to which the existence of charge limits perturbs particle charge distributions, because the relative paucity of free electrons acts to limit particle charge even in the absence of inherent particle charge limits.

To explore this effect we conducted MC simulations for various values of $n_e/n_i$. Figure 5 shows results for 10-nm-diameter silicon particles for the same conditions as in Fig. 2(b), except that $n_e/n_i$ is allowed to vary, and the charge limit is set equal to 14, as given by equation (3). We consider three values of $n_e/n_i$: 1, 0.25 and 0.01.

For the case $n_e = n_i$, without charge limits the charge distribution is well represented by the Matsoukas and Russell expression [7, 8], with an average charge of $-24.5$. However, accounting for the charge limit, which here equals $-14$, the true charge distribution must obviously be completely different. With electrons as abundant here as positive ions, particles bunch into the charge state given by $-q_{\text{lim}}$. In this case, the simulation indicates that 80.4% of the particles ex-
ist in this charge state, with the other 19.6% of particles distributed into charge states that are less negative.

For the case $n_e/n_i = 0.25$, the reduced abundance of electrons relative to ions causes the distribution without charge limits to become less negative. The peak in this distribution is seen to be located at a charge of $-15$, close to the charge limit of $-14$. The distribution with charge limits is now seen to be rather close to that without charge limits, except that the former is truncated at the value of the charge limit. The bunching of particles into the charge state given by $-q_{lim}$ is now less pronounced, with only 36.2% of the particles located in this charge state, according to the MC simulation results.

When electrons are more severely depleted, the existence of charge limits begins to make little difference in the charge distribution, because there are not enough electrons per positive ion to charge particles to their limit. Thus, for the case $n_e/n_i$, the distributions with and without charge limits are seen to be virtually identical. Even without charge limits the distribution is shifted far toward positive values—indeed, a non-negligible fraction of the particles are neutral.

Figure 5. Results of Monte Carlo simulations for 10-nm-diameter silicon particles, with or without charge limits, for same plasma conditions as in Figure 2 except that $n_e/n_i$ is varied. Solid line in bottom graph shows Gaussian distribution predicted without charge limits, equation (7).
or positively charged—and effectively no particles are charged to their limit. Therefore the existence of charge limits in this case makes negligible difference in the charge distribution. However, it is worth noting that in this case the charge distribution deviates significantly from a Gaussian distribution, with or without charge limits, because the assumptions involved in the derivation of equation (7) are no longer satisfied [7]. Additionally it should be noted that in this case, where the requirement of overall charge quasi-neutrality implies that the density of negative charge carried by dust particles is almost as large as the positive ion density, the assumption that particles are electrically isolated from each other is likely to be violated, leading to particles being less negatively charged, on average, than in the isolated-particle case [20].

6. Effect of ion mass

From equation (9), without accounting for the existence of charge limits, the average particle charge \( \bar{q} \propto \ln(m_e / m_i)^{1/2} \). Therefore, even though the ion mass makes no difference in the value of the charge limit, it can make a difference in the extent to which the existence of charge limits perturbs the charge distribution. For example, Fig. 6 shows stationary particle charge distributions calculated by MC simulations for 10-nm-diameter silicon particles (\( q_{\text{lim}} = 14 \)) with either Ar\(^+\) or H\(^+\) as the positive ion. Plasma conditions are otherwise the same in both cases: \( n_e / n_i = 1 \), \( T_e = \text{eV} \), \( T_i = 300 \text{ K} \). The higher mobility of H\(^+\) compared to Ar\(^+\) causes the charge distribution to be shifted toward positive values in the hydrogen case. As seen in Fig. 6, both distributions are truncated at the charge limit, but the truncation is more severe in the argon case. The two distributions are still dissimilar, but the existence of the charge limit at –14 now causes the average charge to be much closer in the two cases than if the limit did not exist.

7. Effect of charge limits on temporal behavior of charge distribution

To this point we have considered the effect of charge limits only on the stationary distribution of particle charge. We now consider two aspects of the effect of charge limits on the temporal behavior of the particle charge distribution: the time required to reach a steady-state distribution, and the power spectrum of charge fluctuations.
7.1 Effect of charge limits on time required to reach steady-state charge distribution

The existence of charge limits reduces the time required to reach a stationary charge distribution. To explore this effect we used MC simulations assuming various charge limits, with the initial condition that all particles are neutral. Figure 7 shows the temporal variation of average charge for charge limits of 5, 15, or \( \infty \) (no charge limit), assuming the same conditions as in Fig. 2. The time required for the average charge to reach steady state is one measure of the time required for the charge distribution to reach steady state. From Fig. 7, we see that this time equals \(~100\) ms for the case without charge limits, but only \(~1\) ms if the charge limit equals 15 and 0.1 ms if the charge limit equals 5. This behavior is straightforward to understand, as the more severe the charge limit the higher the fraction of particles that bunch into the charge state given by the limit, and the more closely, and rapidly, the average charge is given by the value of the limit itself. Indeed, aside from the much more rapid approach to steady state, charge limits are seen to strongly reduce fluctuations in average charge, which persist up to long times for the case without charge limits.

Figure 6. Particle charge distributions for 10-nm-diameter silicon particles (\(d_{\text{lim}} = 14\)), with either \(\text{Ar}^+\) or \(\text{H}^+\) as the positive ion. Conditions: \(n_e / n_i = 1\), \(T_e = 3\) eV, \(T_i = 300\) K.
Effect of charge limits on power spectrum of charge fluctuations

To characterize the nature of charge fluctuations in the presence of charge limits, we calculate the power spectrum of the fluctuations, defined by

\[ P(\omega) = \left| F[q(t)] \right|^2, \tag{22} \]

where \( \omega \) is the fluctuation frequency, and \( F \) is the Fourier transform of \( q(t) \), given by

\[ F(\omega) = \int_{-\infty}^{\infty} q(t) e^{-i2\pi\omega t} dt. \tag{23} \]

Figure 8 shows the charge fluctuation power spectrum for 5-nm-diameter particles both without charge limits (left) and with a charge limit of \(-5\) (right). In both cases we calculate the power spectrum after the charge distribution has reached steady state, so that the change in the frequency spectrum is due to the difference in charge distributions not the difference in charging.

Figure 7. Temporal evolution of average charge for 10-nm-diameter particles in an argon plasma with \( n_e/n_i = 1, T_e = 3 \text{ eV}, T_i = 300 \text{ K}. \)
times between the two cases. The existence of a charge limit is seen to greatly increase the relative importance of high-frequency versus low-frequency fluctuations. This contrasts with the study of Cui and Goree [6], who neglected charge limits and found that the charge fluctuation power spectrum is dominated by low frequencies, and that higher frequency fluctuations decay as $\omega^{-2}$. It should be noted that this effect is particularly pronounced for very small values of the charge limit, and becomes less pronounced at larger values of the charge limit.

![Figure 8](image.png)

Figure 8. Power spectrum of charge fluctuations for 5-nm-diameter particles in an argon plasma with $n_e / n_i = 1$, $T_e = 3$ eV, $T_i = 300$ K. (a) Without charge limits; (b) with a charge limit of $-5$.

8. Conclusion

Previous studies that have derived expressions for the stationary particle charge distribution function in dusty plasmas have neglected the fact that the amount of charge a dust particle can hold is limited. In this paper we derived an analytical expression for the stationary charge distribution that accounts for the existence of charge limits. This derivation was obtained by suitably modifying the Matsoukas and Russell expression [7, 8] for the charge distribution that results from OML charging without charge limits. Values of average charge and variance of the distribution with charge limits were also obtained. The validity of the analytical expression was tested using MC simulations of particle charging. Agreement between the analytical expression and numerical simulations was found to be excellent over a wide range of charge limits, except for
the value of the standard deviation of the distribution in cases where the charge limit is very small. In this regime, the assumptions of the Matsoukas and Russell expression that lead to a Gaussian charge distribution are known to be violated. Even here, however, the analytical expression produces a quite reasonable approximation to the “true” distribution (under the assumption of OML charging) predicted by the MC simulations.

For cases where electron depletion by attachment to dust particles causes $n_e / n_i$ to become small, it was shown that the existence of charge limits becomes less important, because the paucity of electrons relative to ions can cause particles to become much less negatively charged than their charge limit allows. It was also shown that ion mass plays an important role in the relative importance of the existence of charge limits, as lighter ions cause the charge distribution to shift toward less negative values.

The effect of charge limits on the temporal behavior of the charge distribution was examined. It was shown that the existence of charge limits can have a strong effect on the time required to achieve a steady-state charge distribution, with smaller charge limits promoting a more rapid approach to steady state. Additionally, examination of the power spectrum of charge fluctuations indicates that the existence of charge limits increases the relative importance of high-frequency versus low-frequency charge fluctuations, especially when the charge limit is very small.

It should be noted that the theory and numerical simulations presented here consider only electron and ion attachment, following OML theory, with a sharp cutoff at the particle charge limit. Thus, while electron field emission is discussed as one mechanism that limits particle charge, it is not otherwise taken into consideration as affecting the charge distribution. In principle electron field emission could be included in the MC simulations. However this would require knowledge of the particle-charge-dependent field emission current, which would likely be material-dependent as well. Regrettably this effect remains poorly understood, and quantitative models are lacking. Nevertheless, one can anticipate that the emissive current would be greatest at the charge limit itself, and would decrease for smaller negative charge values. Similarly, as a particle’s electron affinity decreases as the particle charge approaches the charge limit, it is possible that the electron sticking coefficient would become smaller than unity even though attaching an electron does not exceed the charge limit. Either of these effects, if important, could qualitatively alter the results presented here, as the charge distribution would not bunch as strongly.
into the charge state corresponding to the charge limit. This therefore suggests possible future work on charge distributions with particle charge limits.

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Appendix

All integrations needed in this paper are calculated using the following integrals:

\[ I_1 = \int \exp \left[ -\frac{(x - \bar{x})^2}{a} \right] dx, \quad (A-1) \]

\[ I_2 = \int x \exp \left[ -\frac{(x - \bar{x})^2}{a} \right] dx, \quad (A-2) \]

\[ I_3 = \int (x - b)^2 \exp \left[ -\frac{(x - \bar{x})^2}{a} \right] dx. \quad (A-3) \]

The values of these integrals are as follows:

\[ I_1 = \frac{1}{2} (\pi a)^{1/2} \text{erf} \left( \frac{x - \bar{x}}{a^{1/2}} \right) + C, \quad (A-4) \]

\[ I_2 = I_1 \bar{x} - \frac{1}{2} a \exp \left[ -\frac{(x - \bar{x})^2}{a} \right] + C, \quad (A-5) \]

\[ I_3 = I_1 \left[ \frac{a + 2(\bar{x} - b)^2}{2} \right] - \frac{1}{2} a \exp \left[ -\frac{(x - \bar{x})^2}{a} \right] (\bar{x} - 2b + x) + C, \quad (A-6) \]

where \( C \) is the constant of integration.
References